Geographically Weighted Regression (GWR): A Better Spatial Technique to Model Sediments Quality of Lake Okeechobee

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Outline

I. Background
II. Objectives
III. Methodologies
IV. Data exploration
V. GWR models for total phosphorus (TP)
VI. Conclusions

I. Background

- Lake Okeechobee is the largest lake in the southeastern US with an area of 1730 km^2 and an average depth of 2.7 m;
- Lake Okeechobee has excessive phosphorus loads:
  - TP accumulated in mud sediments (44% of lake bottom)
  - Increased TP concentration in water column
  - Increased algal blooms
- It is critical to track the nutrient distribution and its changes over-time

II. Objectives

- Develop optimal spatial models for TP in the lake sediments;
- Examine the spatial changes of TP (1988-2006);
- Calculate the amount of nutrients (TP), its spatial-temporal changes, and identify the hotspots.

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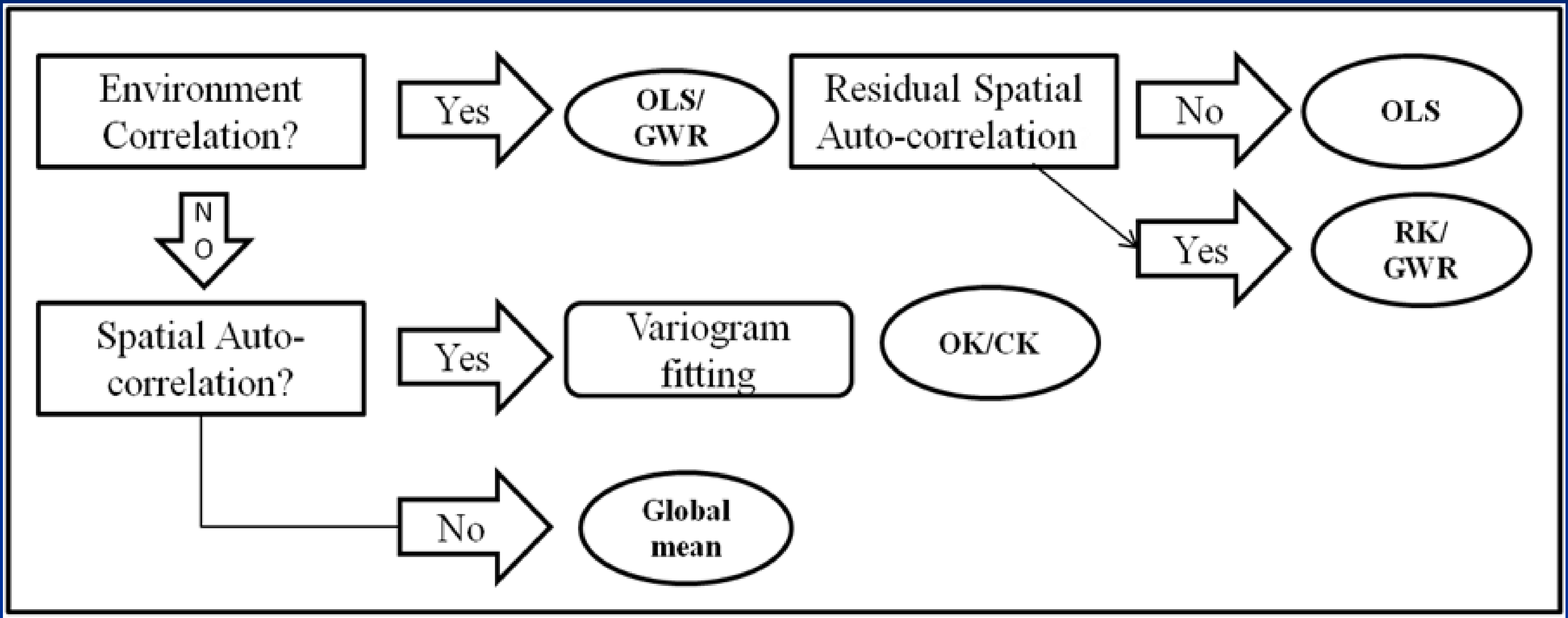
III. Methodologies

- Changes of spatial modeling techniques
- Model selection & model accuracy
- Geographically Weighted Regression (GWR)
  - Weighting functions & bandwidth
  - Optimal bandwidth & nearest neighbors
  - Model performance

Development of Spatial Modeling Techniques

- Great efforts have been made to develop generic and robust spatial interpolation techniques for many years;
- Kriging and its variants (from 1950s -)
- Environmental correlation (Regression) (1990s -)
  - Development of GIS and remote sensing technologies
- Hybrid models
  - Ancillary data is used to improve spatial prediction;
  - Good for non-stationary data;
  - Examples:
    - Regression-Kriging (RK) (late 1990s-)
    - Spatial regression (GWR) (2000s-)

Geostatistical Model Selection Procedure



(OLS: ordinary least square, OK: ordinary kriging; CK: Co-kriging; RK: regression-kriging and GWR: geographically weighted regression)

Model Accuracy and Comparison

- Use objective criteria to evaluate model errors, robustness and reliability
- Mean error:

$$ME = \frac{1}{l}\sum_{i=1}^{l}[\hat{z}(s_i) + Z^*(s_i)];\ E\{ME\} = 0$$

- Root mean square error (RMSE):

$$RMSE = \sqrt{\frac{1}{l}\sum_{i=1}^{l}[\hat{z}(s_i) - Z^*(s_i)]^2}\ ;\ E\{RMSE\} = \sigma(h = 0)$$

- Scatter plots of observed vs. predicted values

What is GWR?

- GWR analyzes spatially varying relationships between the target variable and ancillary variables;
- Developed by Fotheringham and Brunsdon (2000, 2002)
- Based on the "First Law of Geography": everything is related with everything else, but closer things are more related (Tobler 1970);
- βks vary from location to location, and the interpolation can be expressed as:

$$y_i = \beta_0 (u_i v_i) + \sum_{k=0}^{n} \beta_k (u_i v_i) x_{ik} (u_i v_i) + \varepsilon_i$$

$(u_i v_i)$ is the coordinates of the *i*th observation

$\beta_k (u_i v_i)$ is a realization of the function $\beta_k (u, v)$ at point *i*

GWR vs. OLS

- OLS:
  - Global regression model
  - One equation, calibrated using data from all features
  - Relationships are fixed
- GWR:
  - Local regression model
  - One equation for every feature, calibrated using data from nearby features
  - Relationships are allowed to vary across the study area

GWR Weighting Schemes and Bandwidths

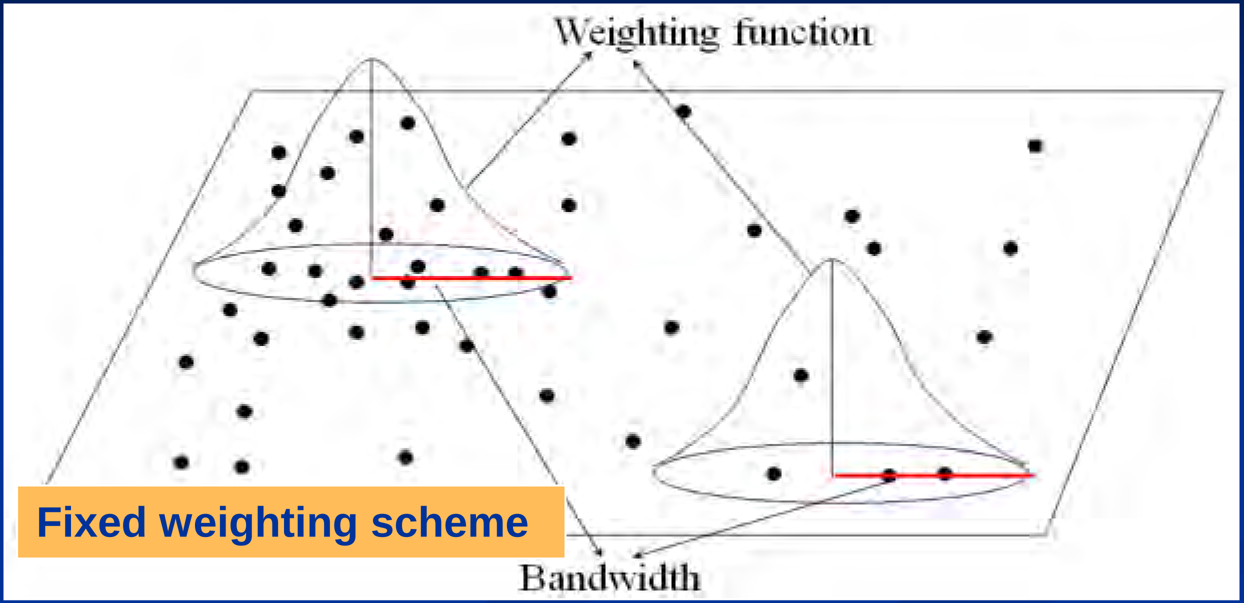
- Model parameters depend on weighting function and bandwidth
- Gaussian scheme:

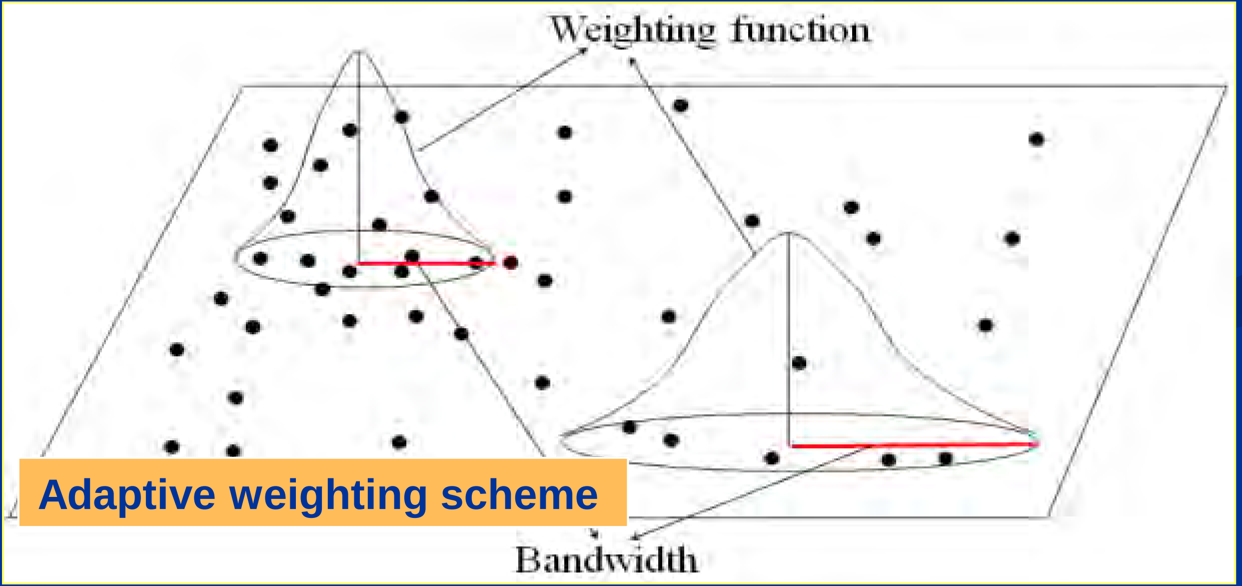
$$w_{ij} = \exp\left[-\frac{\frac{d_{ij}^2}{h^2}}{2}\right]$$

- Bi-square scheme:

$$w_{ij} = \left[1 - \left(\frac{d_{ij}^2}{h^2}\right)\right]^2 \quad \text{if } d_{ij} < h$$

$$= 0 \quad \text{otherwise}$$





Optimal Bandwidth/Neighbor Selection

- An optimal bandwidth (or nearest neighbors) can be selected by satisfying either
  - the least cross-validation (CV) score:

$$\mathrm{CV} = \sum_{i=0}^{n} (\mathrm{y_{i}} - \mathrm{y_{j \neq i}})^2$$

  - or least Akaike Information Criterion (AIC):

$$\mathrm{AIC_c} = 2n\log_e(\hat{\sigma}) + n\log_e(2\pi) + n\{\frac{n+\mathrm{tr}(\mathbf{S})}{n-2-tr(\boldsymbol{S})}\}$$

GWR Model Performance & Diagnosis

- The corrected AICc is the measure of goodness of fit for GWR models;
- AICc provides a measure of the information distance between the fitted model and the unknown ‘true’ model;
- AICc can be used to compare the global OLS model with a local GWR model;
- Other model performance indicators:
  - Sum of the squared residuals (SSR)
  - Sigma , and
  - R^2

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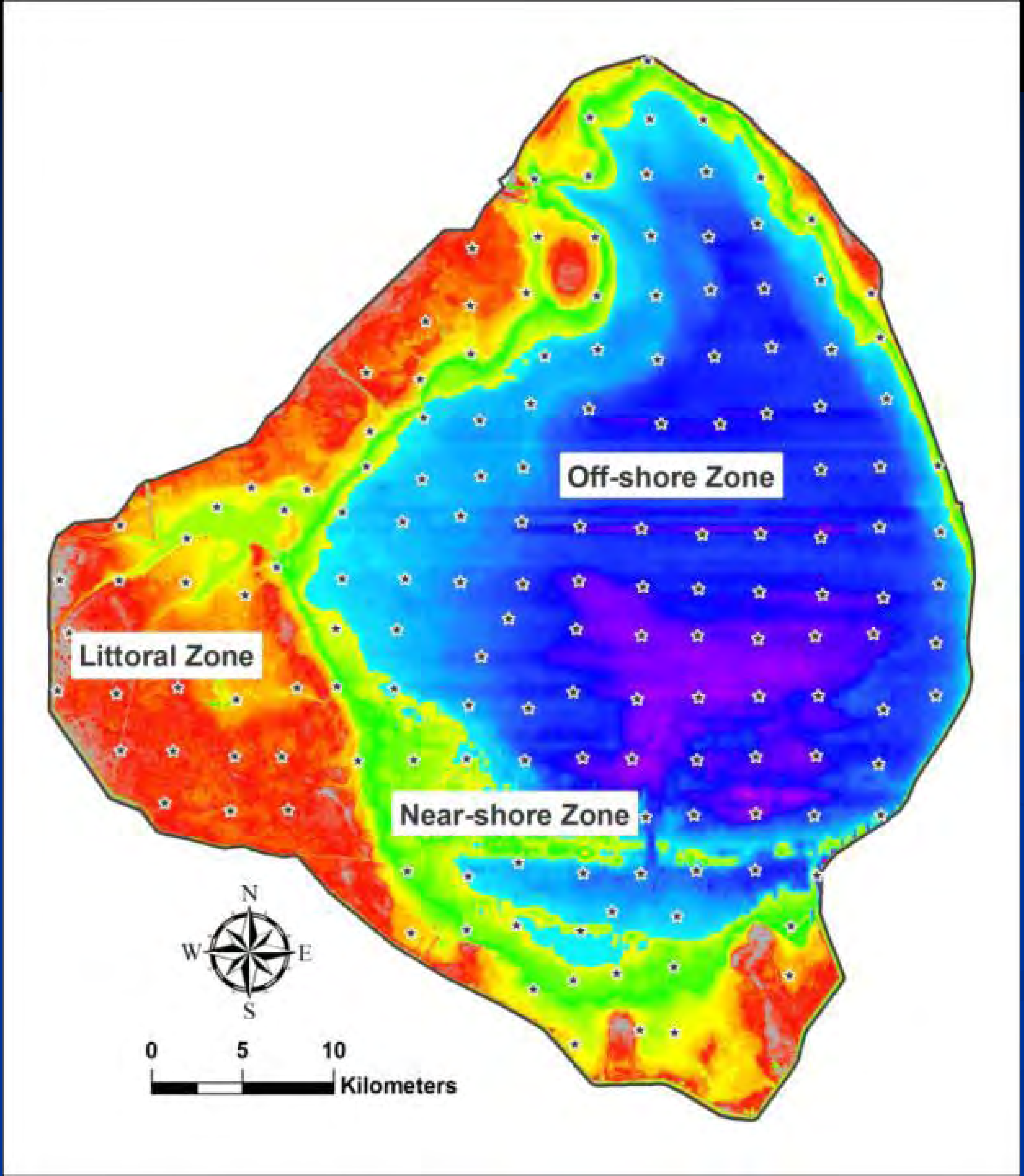
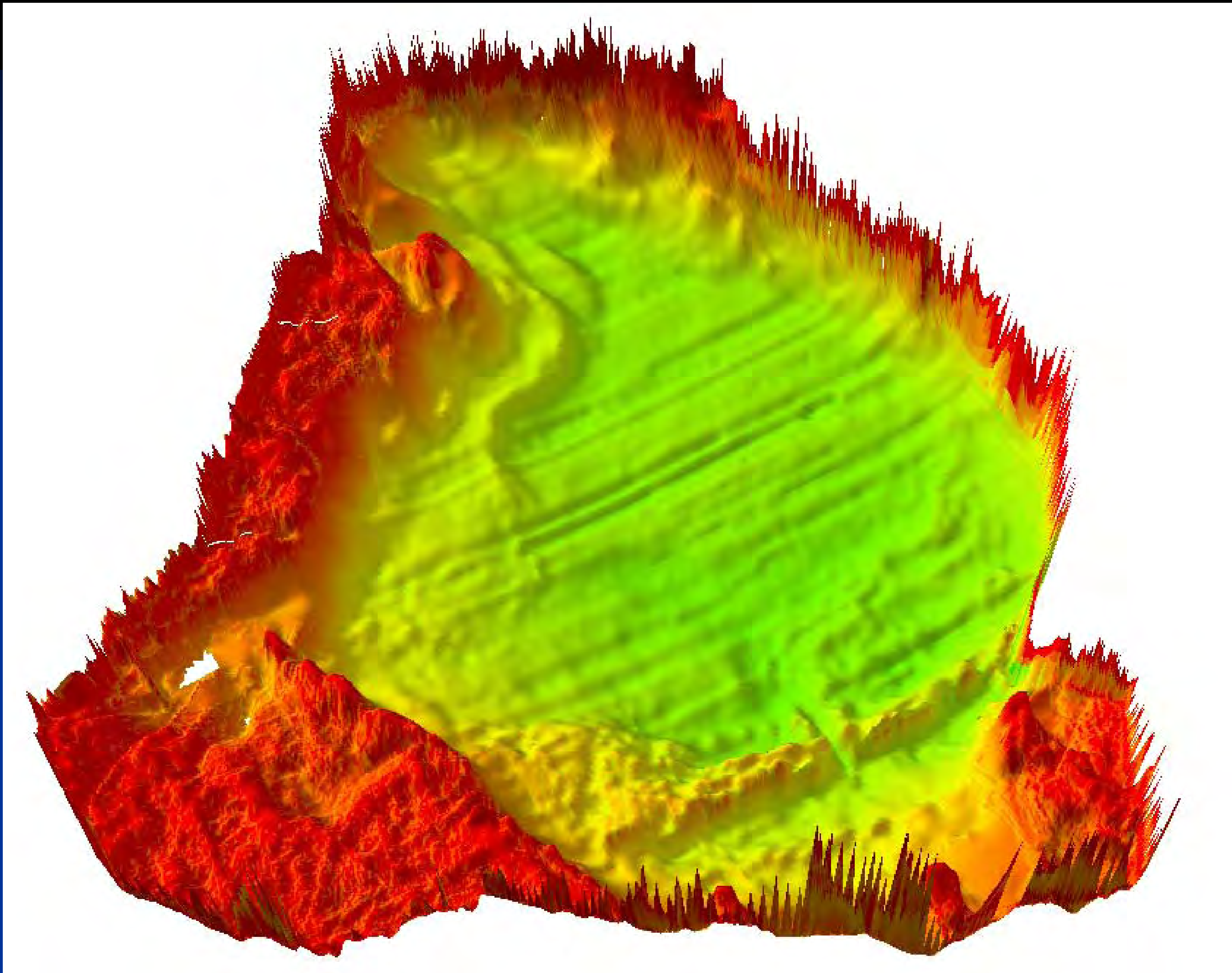
IV. Data Exploration

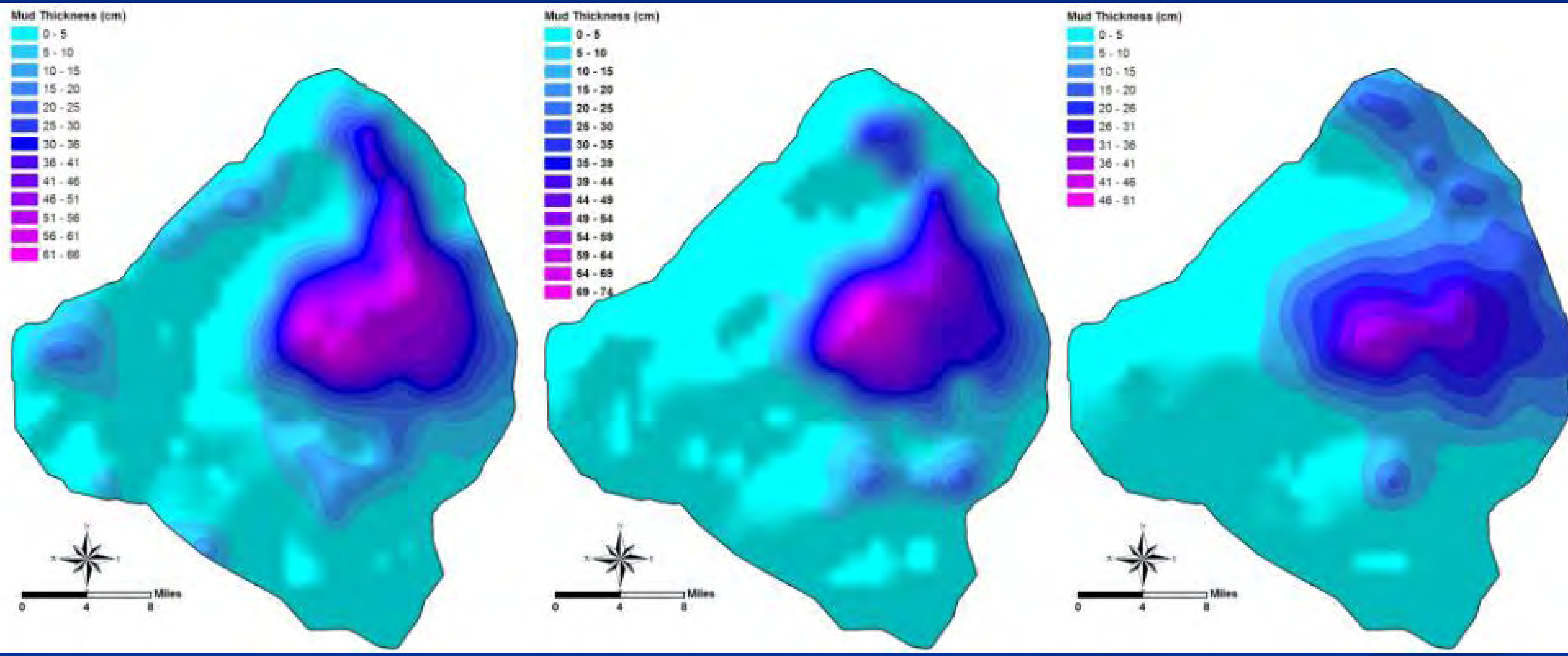
- Sediments Data
  - 170 sediment core samples taken in 1988, 1998 and 2006
  - Mud thickness and nutrients (TP, TN, TC, Fe, Ca, etc)
  - Mud density
- Digital Elevation Model
  - Lake bathymetry data (2008), and LIDAR (2007-2008)
  - Bathymetry & LiDAR data integration
- Data summarization and characteristics
  - Distribution
  - Trends
  - Correlations



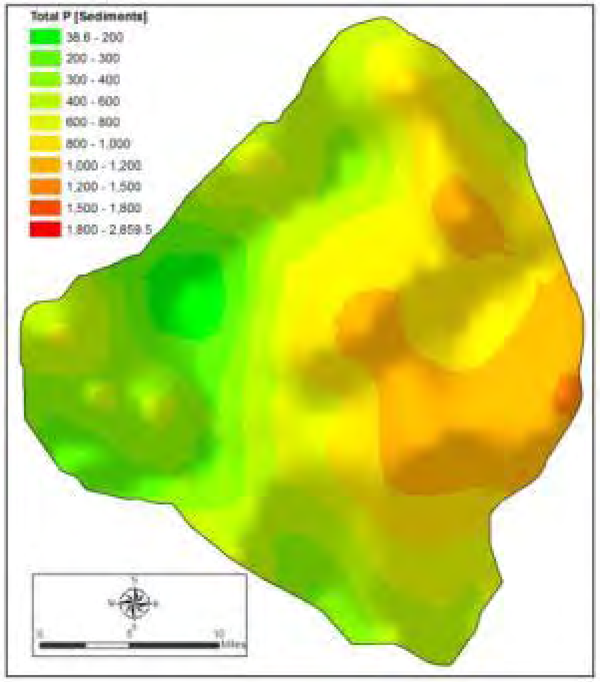
Bathymetry, eco-zones and sample sites Lake Okeechobee

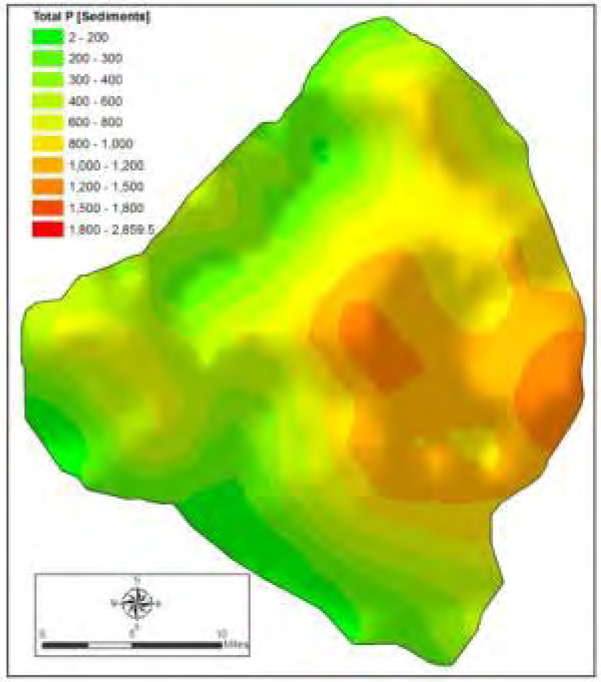


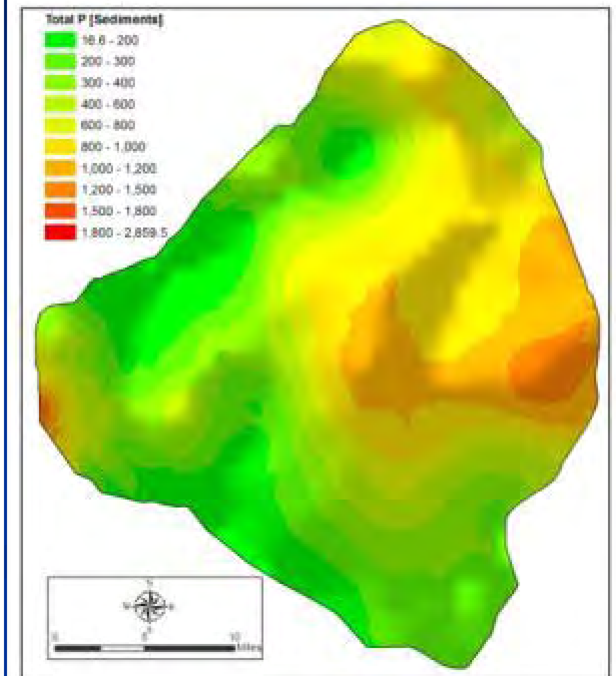
Mud Thickness Variation Using OK



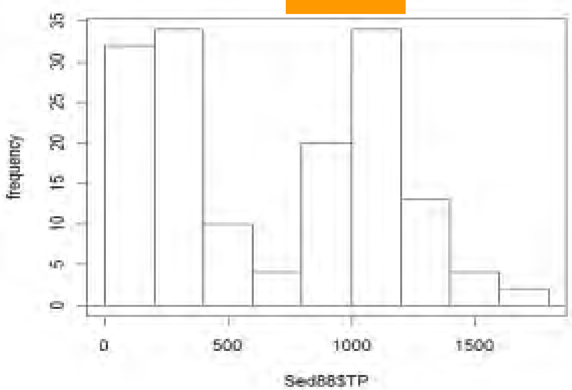
TP Histograms and Maps Using OK



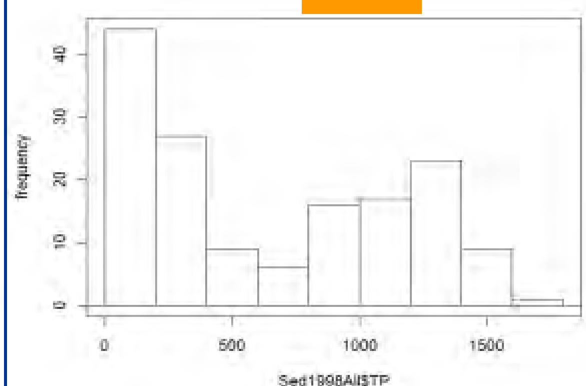




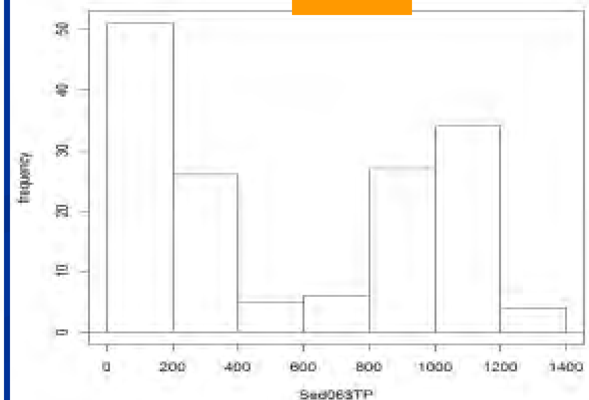
1988



1998

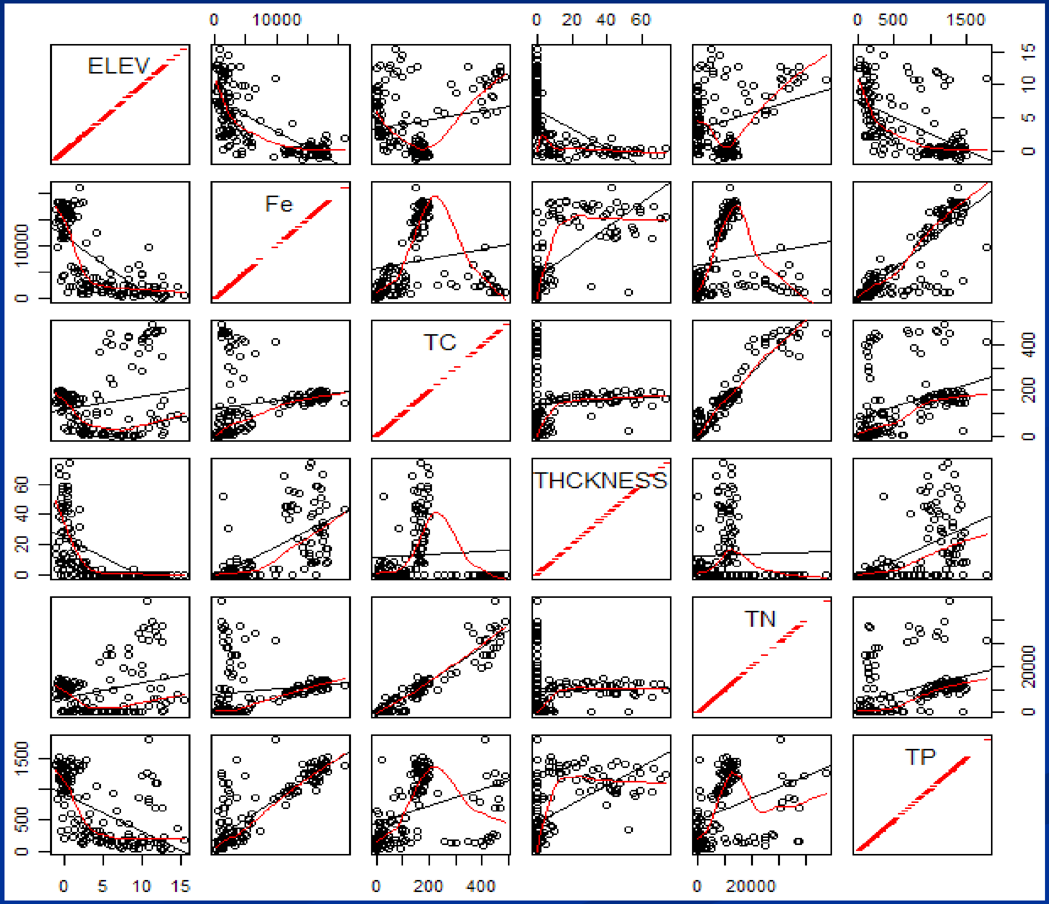


2006



Explore Relationship Using Scatter Plot Matrix

- TP
  - Strong positive correlation with Fe
  - Weak positive correlation with mud thickness and TC
  - Weak negative with site elevation
- All these similar correlations exit in all 3 data sets



1998 data

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V. GWR Model Calibration & Validation

- First identify the best OLS regression models using the ArcGIS OLS Tool;
- Then perform GWR analysis using the ArcGIS GWR Tool;
- Compare the fit/performance of the OLS & GWR models using R^2 and AICc;
- Select the best GWR sub models for validation.

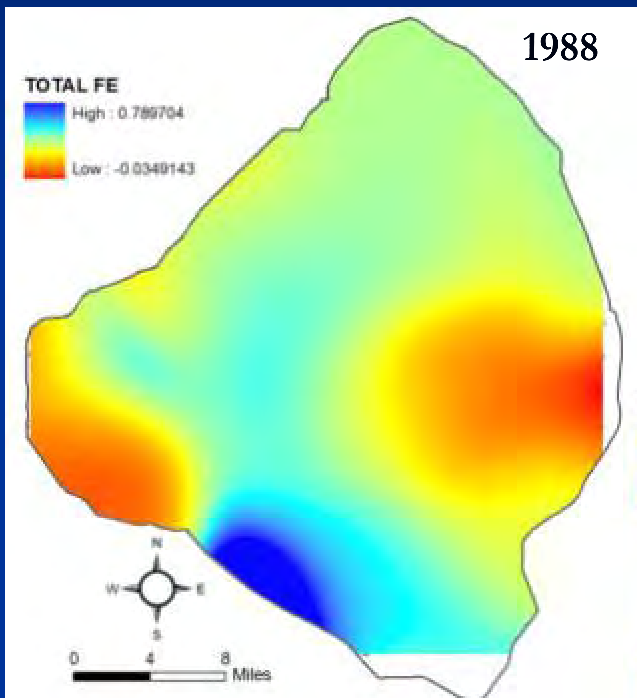
Statistically Significant Regression Models

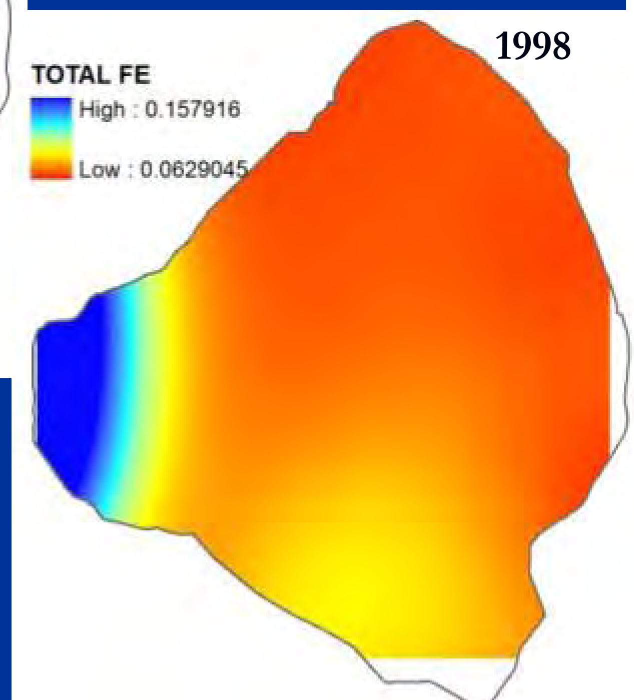
- Best regression models identified using OLS
  - TP vs. Total Fe
  - TP vs. Mud Thickness (Th) and Elevation (Elev)
- Perform GWR model analysis for these models

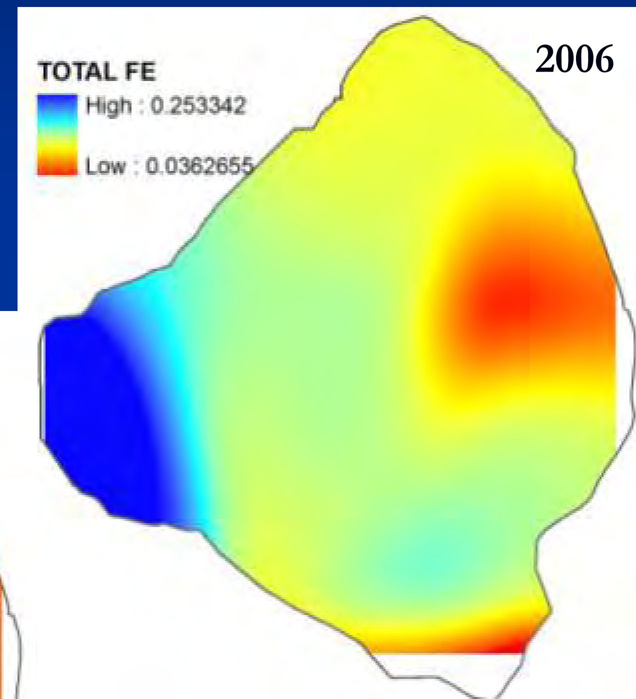
GWR (TP vs. Fe) Model

	GWR Models	Adaptive CV	Adaptive AIC	Fixed CV	Fixed AIC
2006	Neighbours	32	38	**25048.46**	28060.8468
	ResidualSquares	1474546	1559401.91	**1370525**	1479068.08
	Sigma	125.7934	127.1493	**125.8744**	127.5319
	AICc	1442.359	1442.0453	**1445.878**	1445.1828
	R^2	0.9322	0.9283	**0.937**	0.932
	R^2Adjusted	0.9178	0.916	**0.9176**	0.9155
1998	Neighbours	114	46	2583312.66	**38985.12**
	ResidualSquares	5721282	4772089.89	5851677.26	**4754339**
	Sigma	228.3922	219.3559	228.5793	**217.7004**
	AICc	1566.658	1563.9183	1566.1879	**1560.323**
	R^2	0.818	0.8482	0.8138	**0.8487**
	R^2Adjusted	0.8124	0.827	0.8121	**0.8296**
1988	Neighbours	13	38	**21433.62**	24336.93
	ResidualSquares	2256157	4669867.28	**3403058**	3809536.23
	Sigma	189.9278	220.3011	**205.7668**	210.1373
	AICc	1582.818	1567.6434	**1564.545**	1563.3411
	R^2	0.9067	0.8069	**0.8593**	0.8424
	R^2Adjusted	0.8314	0.7732	**0.8021**	0.7936

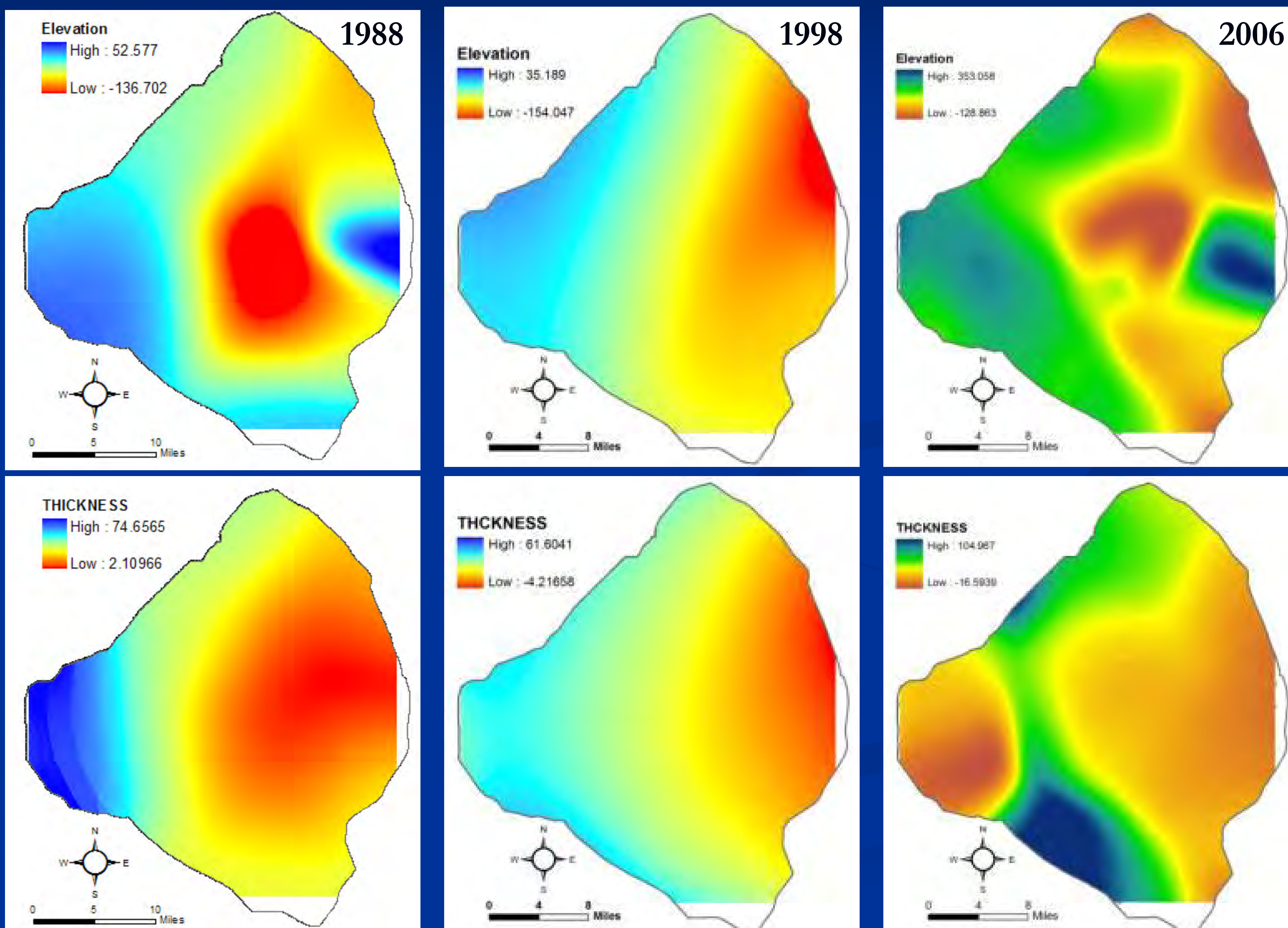
Coefficient Maps of GWR (TP vs. Fe) Model



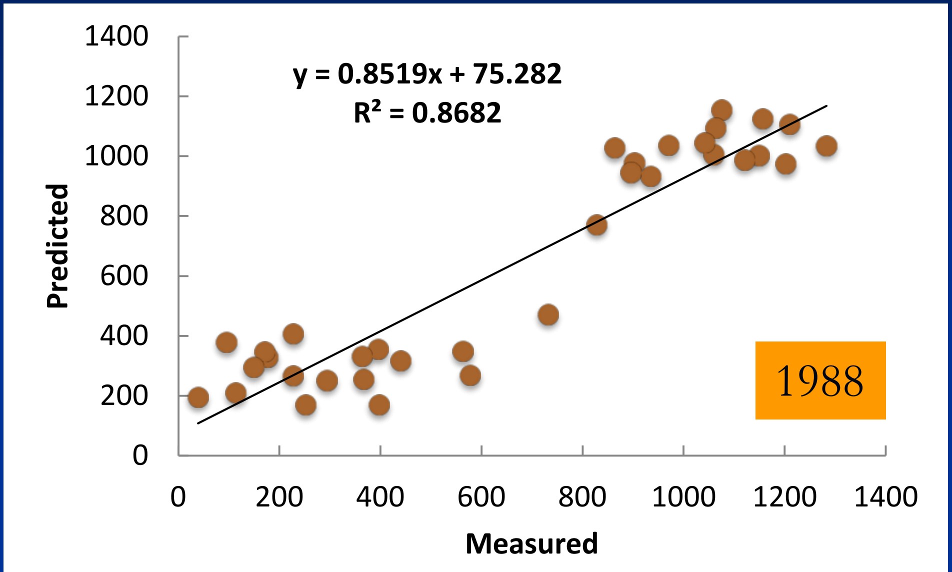




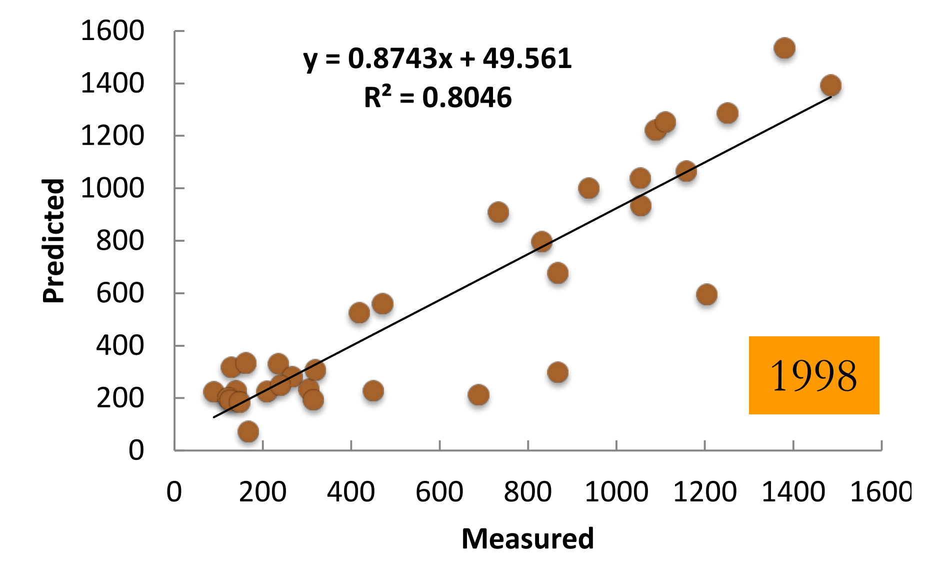
Coefficient Maps of GWR (TP vs. Th & Elev)

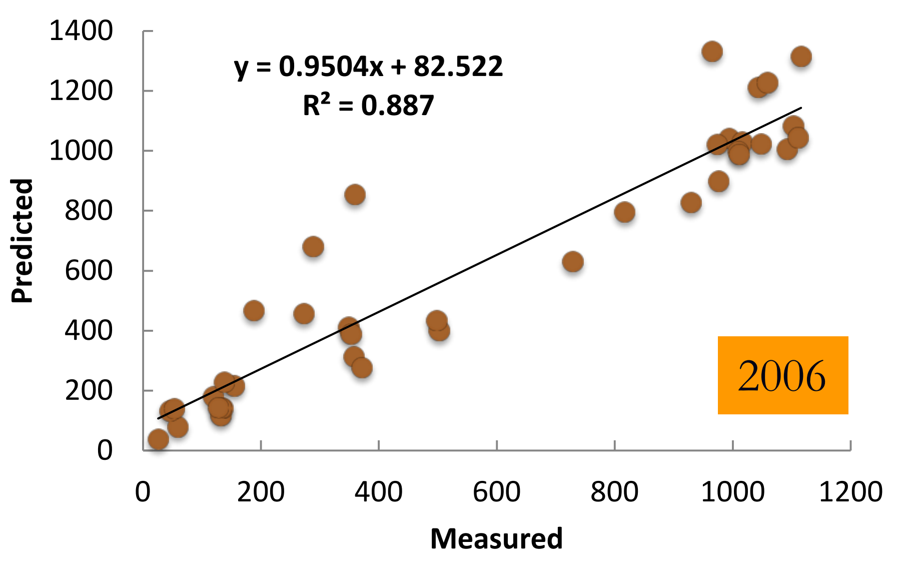


GWR (TP vs. Fe) Validation Results

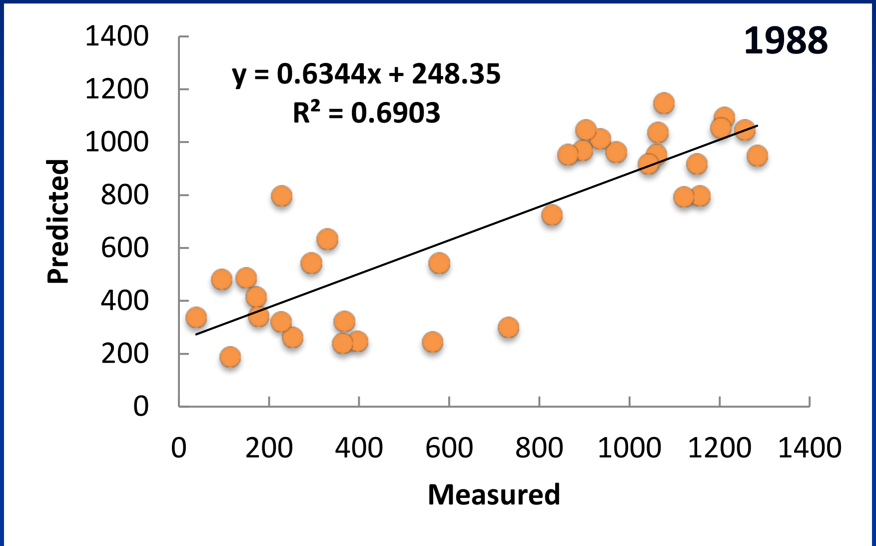


Scatter plots of observed and measured values at validation sites





GWR (TP vs. Th & Elev) Validation Result



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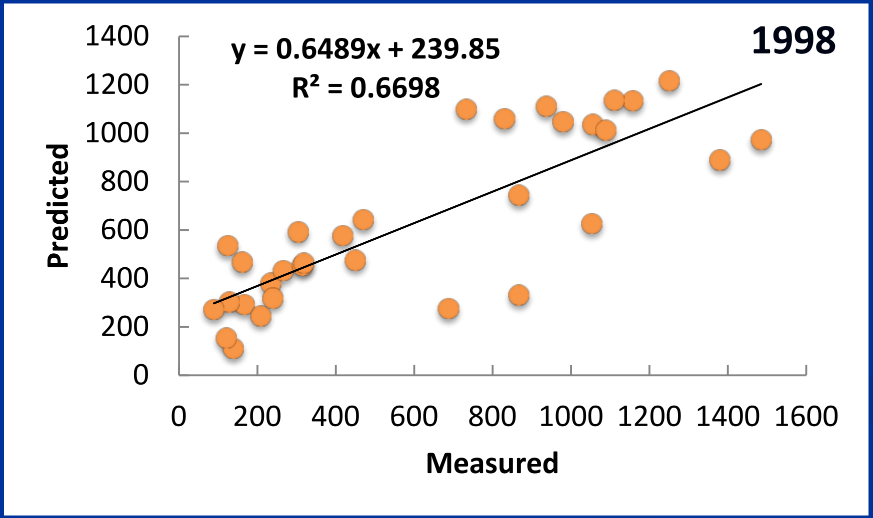
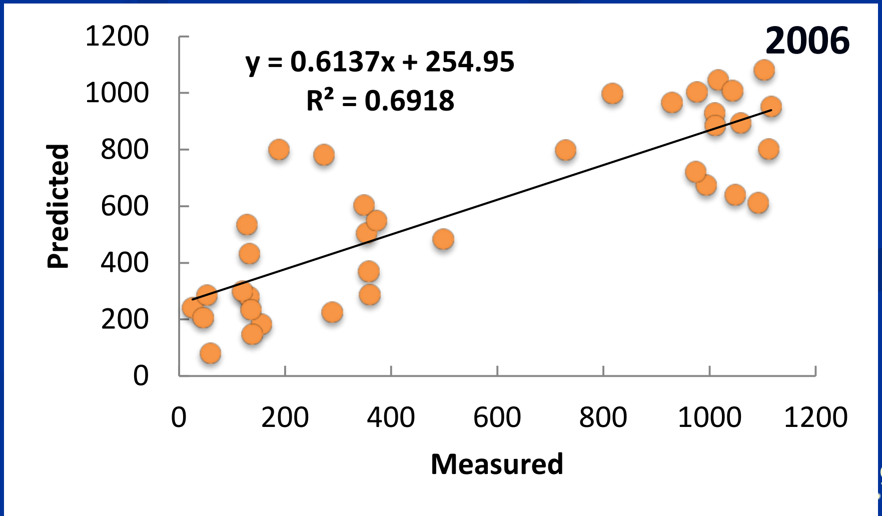
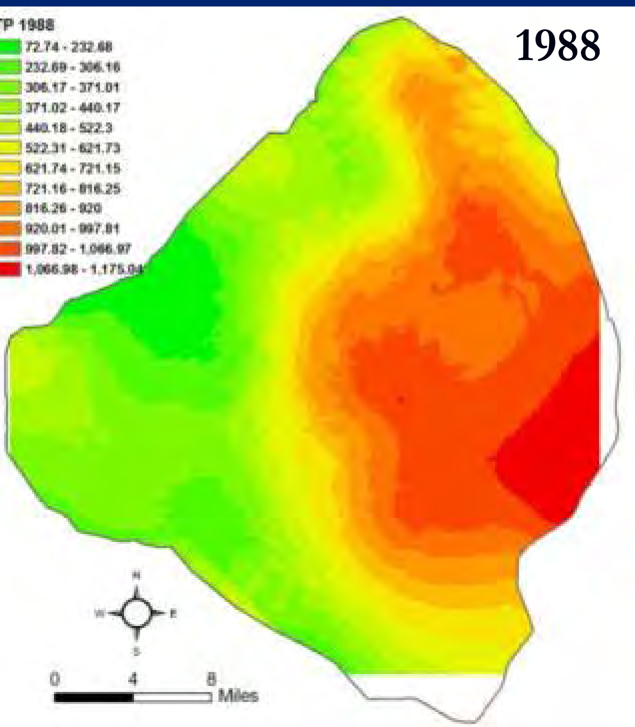


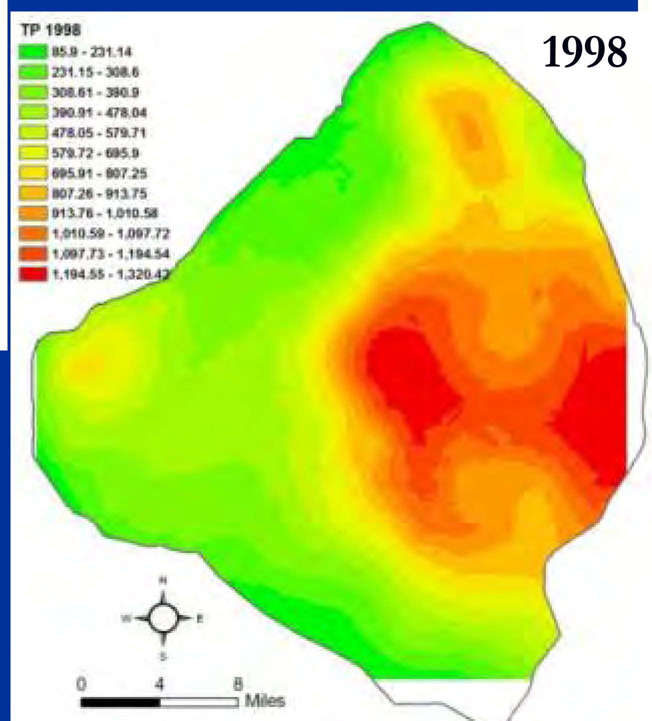


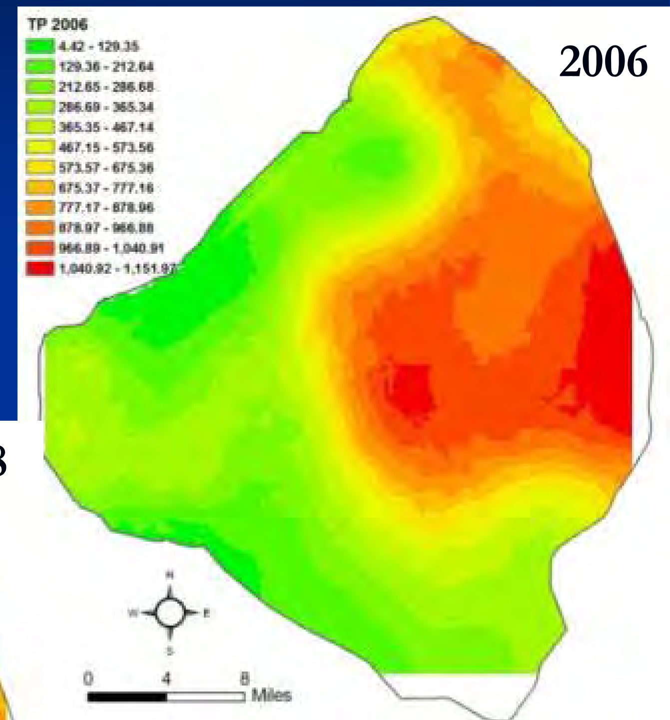
Model Accuracy and Comparison for TP

	Models	OK	OLS (Fe)	GWR (Fe)	OLS (Th, Elev)	GWR (Th, Elev)
2006	Mean Error	-27.4	31.19	-30.19	40.21	20.82
	RMSE	331	169.96	204.61	325.23	307.6
	NRMSE	26.09	13.4	16.13	25.64	24.25
1998	Mean Error	-89.5	136.93	132.81	106.64	100.58
	RMSE	415.1	374.75	359.83	442.98	389.38
	NRMSE	23.18	20.92	20.09	24.73	21.74
1988	Mean Error	-41.7	110.94	109.59	64.15	24.39
	RMSE	402.1	368.36	375.88	371.97	234.34
	NRMSE	0.24	0.22	0.23	0.22	0.14

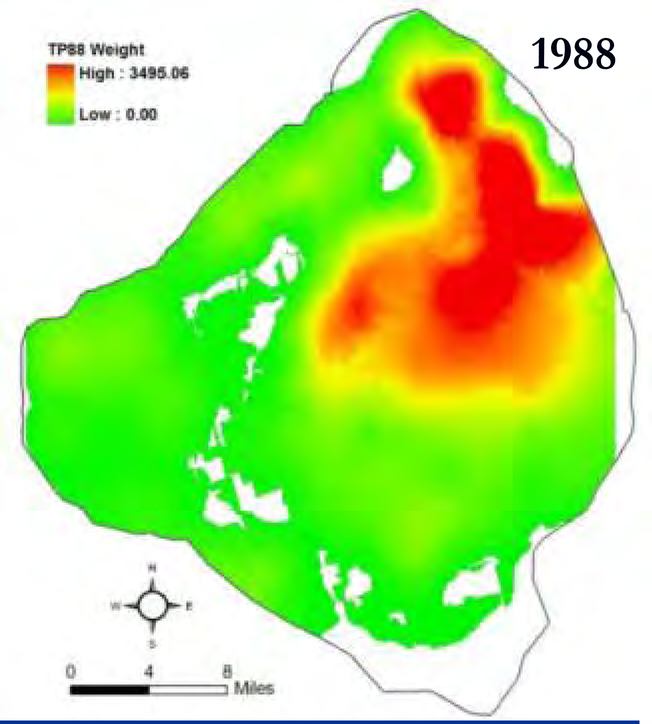
TP concentration Using GWR (TP vs. Fe)

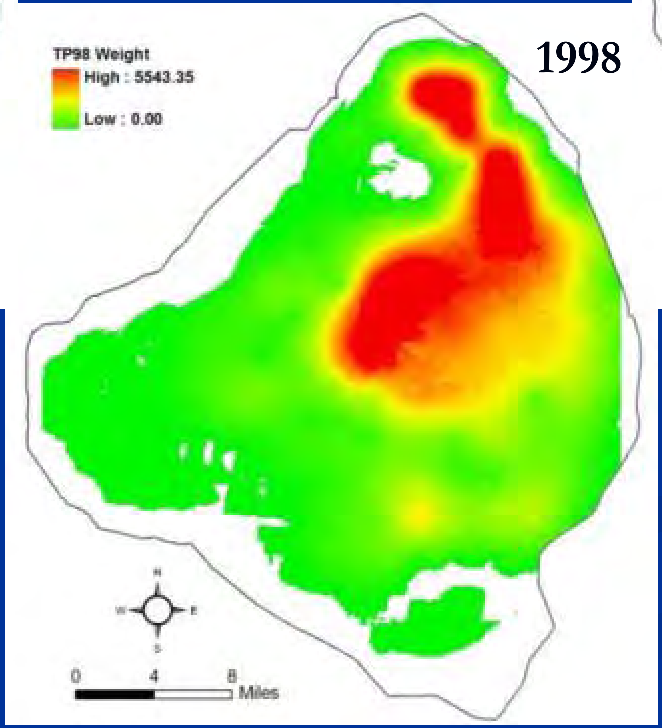


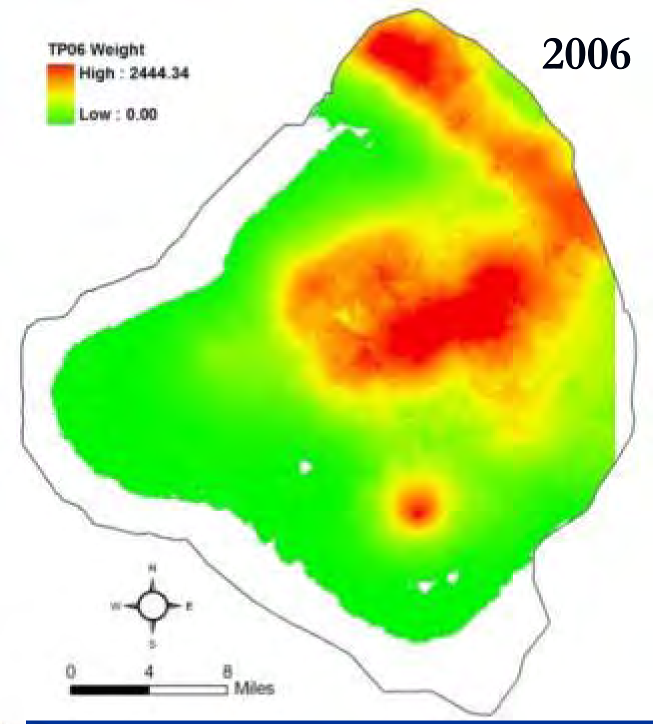




TP Weight Distribution Using GWR (TP vs. Fe)







TP Weights (mt) & Changes

Model	Ordinary Kriging			GWR (TP vs Fe)			GWR (TP vs. Th & Elev)			Average
Year	Weight	Change	%	Weight	Change	%	Weight	Change	%	Weight
1988	42,500			42,000			44,300			42,933
1998	58,900	16,400	39%	60,400	18,400	44%	61,100	16,800	38%	60,133
2006	41,400	-17,500	-30%	41,000	-19,400	-32%	40,100	-21,000	-34%	40,833

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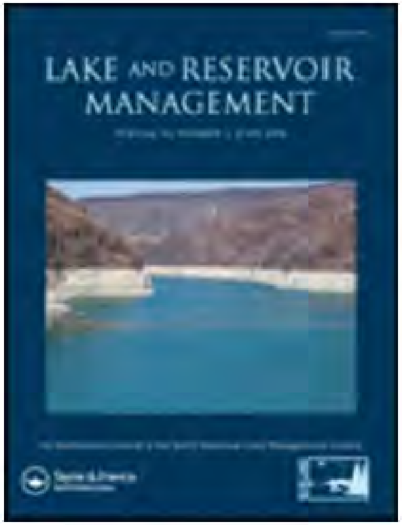
Conclusions

- GWR (TP vs. Fe) and GWR (TP vs. Thick & Elevation) models were most accurate based on RMSE;
- GWR models use both spatial auto-correlation and correlation between TP and independent variables, which improves the model performance;
- Ordinary Kriging (OK) models use only spatial autocorrelation data, which were weak in Lake Okeechobee and produced higher RMSE;

Conclusions (cont.)

- The two best GWR models were used to calculate TP concentrations and TP mass. The TP mass:
  - increased about 38% - 44% from 1988 to 1998 and
  - decreased about 30% - 34% from 1998 to 2006;
- The TP decline from 1998 to 2006 is likely a result of hurricanes that stirred up the sediments which were then discharged from the lake in 2004 and 2005 hurricane seasons.



Geographically Weighted Regression: A Better Spatial Technique to Model Sediment Quality in Lake Okeechobee

QUESTIONS?

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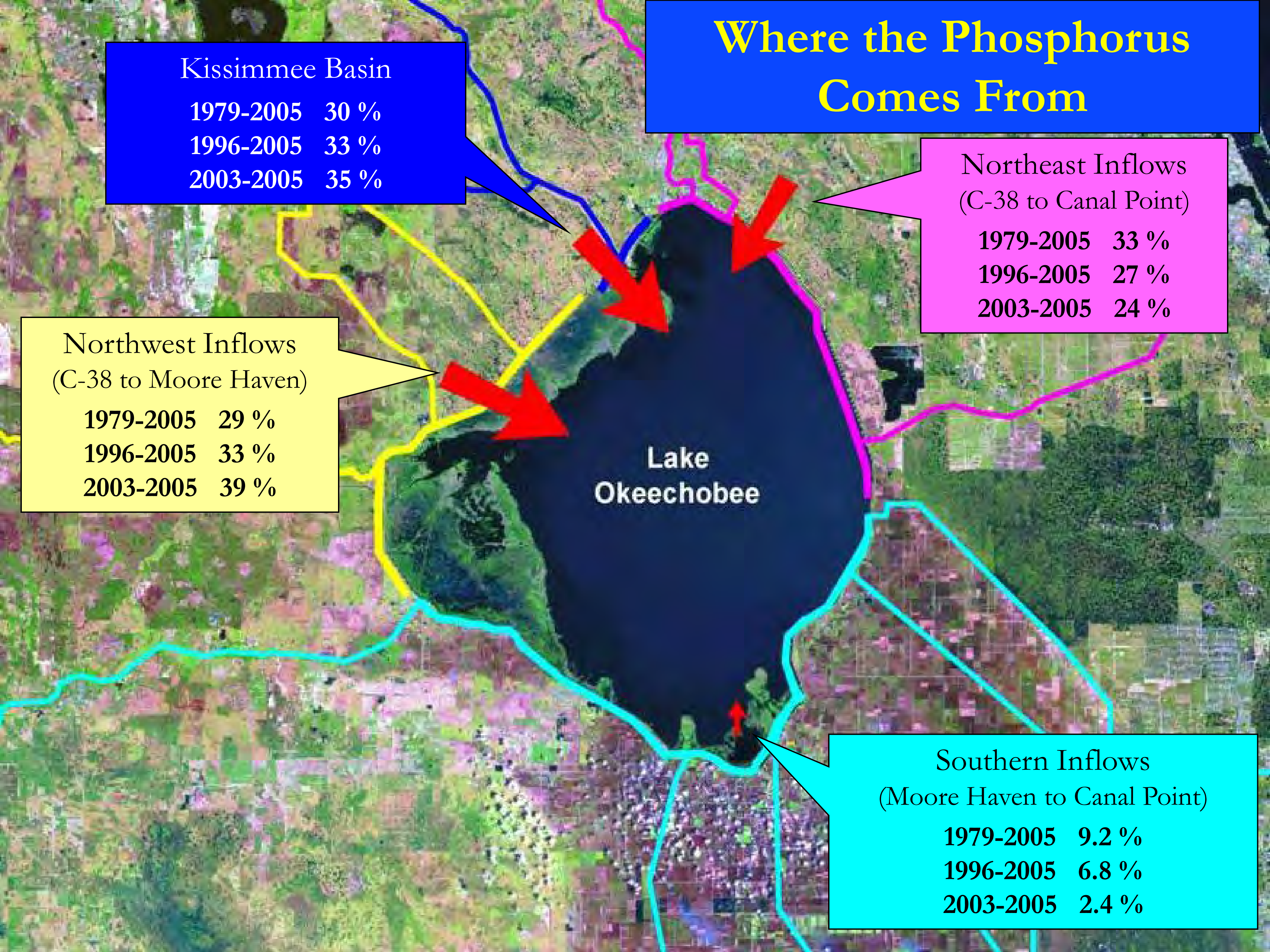
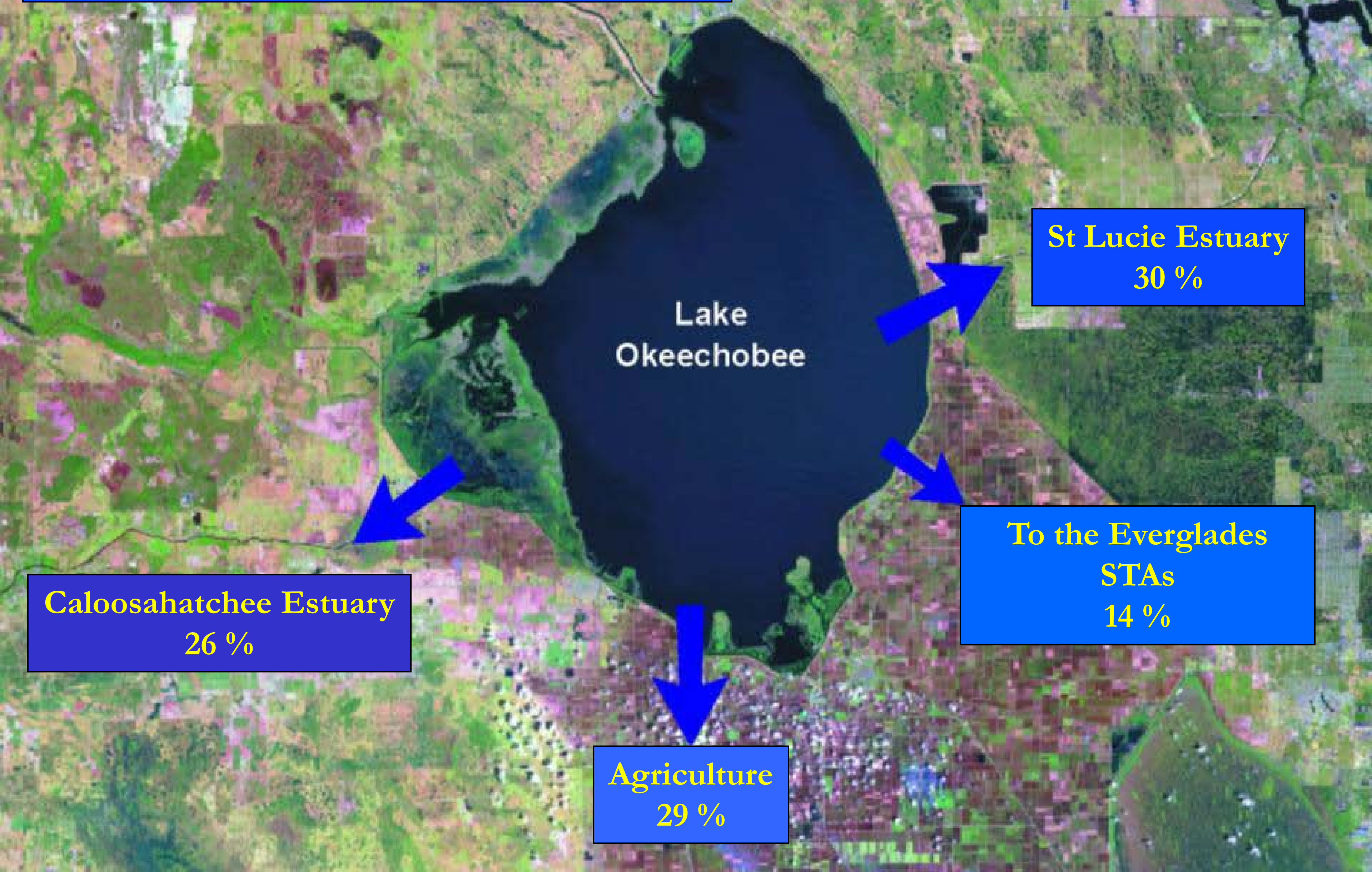

Where the Phosphorus Comes From
Kissimmee Basin
1979-2005 30 %
1996-2005 33 %
2003-2005 35 %
Northeast Inflows
(C-38 to Canal Point)
1979-2005 33 %
1996-2005 27 %
2003-2005 24 %
Northwest Inflows
(C-38 to Moore Haven)
1979-2005 29 %
1996-2005 33 %
2003-2005 39 %
Lake Okeechobee
Southern Inflows
(Moore Haven to Canal Point)
1979-2005 9.2 %
1996-2005 6.8 %
2003-2005 2.4 %

Where the Phosphorus Goes

Based on 1996-2005 data



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